

Midterm Examination 1

Name _____

Student ID Number _____

This exam is **open book, open notes, open computer, open calculator**. An excel file that contains templates for all three problems on the exam has been posted on the course website for you to utilize to answer questions. You are required to submit this template and one Arena file in Moodle under the Midterm Exam 1 Assignment, so leave time for this.

If you prefer to perform all or some calculations by hand, you may do so on this exam paper. A printed hand simulation table is available at the front of the class for Problem 1. The data from the service system of Problem 2 is also available on a separate sheet at the front of the class if you would like extra room for computations . Please staple these pages to the back of your exam if you choose to utilize them.

Please read each of the 3 problems and what is required before beginning to work. You are advised to select the problems you feel most comfortable solving and do them first. If part of a problem requires an answer from a previous part and you could not get the answer to that previous part, assume simple and reasonable numbers for an answer and use those numbers to answer the next part. Show your work whenever you can so that you can receive partial credit if you do something incorrectly. Circle or box your final numerical answers so they clearly stand out. Good Luck!

Problem 1 (25 Points Total). Recently, I was up late watching a Dallas Mavericks game, and when it ended I was hungry and wanted Cookout. My girlfriend drove us to the Capital Blvd. Cookout, which has 2 drive-thru windows that we will consider independent servers which take the next available customer from a single FIFO drive- thru line (see Figure 1). We arrived at exactly 12:30am to find no one in the queue (drive-thru line), but I had to wait until exactly 12:33am to place my order. I ordered, got my food and shake, and then ate in the parking lot before leaving at exactly 12:40am. I saw from my receipt that I was customer number 452. Since I was eating, my girlfriend wrote down the exact time of arrivals and departures to the Cookout drive-thru from 12:30am to 12:40am. The data she recorded is presented in Table 1. 12:00am is considered time 0.

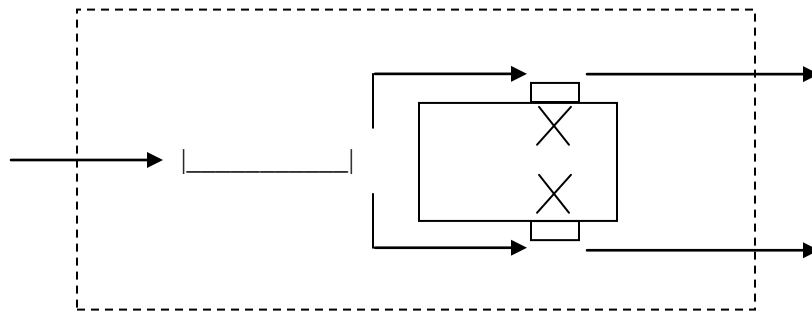


Figure 1. The Cookout Drive-Thru System of Problem 1

Table 1. Data from the Cookout Drive-Thru System of Problem 1

Customer	Arrives at Queue	Departs from Queue	Departs from Server
452 (Me)	30.0	33.0	35.5
453	32.5	35.0	38.0
454	37.0	37.0	39.5
455	39.0	39.0	After We Left

You **MUST** complete the hand simulation table on paper or in the Problem 1 tab of the ISE441Spring2012 - Midterm 1 - Spreadsheet Template.xlsx. The event calendar is optional for time sake but still recommended. You **MUST** also compute the following final values and performance measures and write your answer values for parts (a) - (g) below:

- a. The Total Number of Customers Served (P):

$$P =$$

- b. The Number of Parts that have Passed Through the Queue (Drive-Thru Line) (N):

$$N =$$

- c. The Average Waiting Time in the Queue (Drive-Thru Line) $\left(\frac{\sum_{i=1}^N WQ_i}{N} \right)$:

$$\frac{\sum_{i=1}^N WQ_i}{N} =$$

- d. The Average Total Time Spent at Cookout $\left(\frac{\sum_{i=1}^P TS_i}{P} \right)$:

$$\frac{\sum_{i=1}^P TS_i}{P} =$$

e. The Time-Averaged Number of Customers in the Queue (Drive-Thru Line) $\left(\frac{\int_0^T Q(t) dt}{T} \right)$:

$$\frac{\int_0^T Q(t) dt}{T} =$$

f. The Time-Averaged Total Number of Customers at Cookout $\left(\frac{\int_0^T N(t) dt}{T} \right)$:

$$\frac{\int_0^T N(t) dt}{T} =$$

g. The Utilization of the Drive-Thru Windows $\left(\frac{\int_0^T B(t) dt}{\int_0^T B_A(t) dt} \right)$:

$$\frac{\int_0^T B(t) dt}{\int_0^T B_A(t) dt} =$$

Problem 2 (25 Points Total). Two alternative designs have been proposed for processing customers in a service center. To compare the performance of the two alternative system designs, we generated 10 days of customer traffic, where a day's traffic consists of a new simulated sequence of customer arrival times and the associated service times (in other words, 10 independent replications). For day 1 of simulated operation, we ran the traffic through the system operating under design A to compute the average time in system for all the customers processed on that day/replication. Day 1's traffic is also run through the system operating under design B to compute the average time in system for all the customers processed on that day/replication. The process was repeated for all 10 days of simulated traffic, yielding average time in system (in minutes) for all the customers processed on each day as follows:

Replication	1	2	3	4	5	6	7	8	9	10
Design A Average Time in System	55.2	60.2	45.6	65.2	43.6	38.3	37.5	67.8	88.2	65.7
Design B Average Time in System	50.9	55.2	50.8	57.8	45.6	27.8	34.5	66.9	77.4	46.7

In the Problem 2 tab of the ISE441Spring2012 - Midterm 1 - Spreadsheet Template.xlsx, this data have been provided for you. You may use Excel to do your computations for this problem and you may use formulas in Excel. You **MUST** label your spreadsheet computations and make sure it is easy to determine what numbers in your spreadsheet are... **BE NEAT!** You may also perform calculations by hand on this exam paper or on the separate sheet with this data on it at the front of the class. Just remember to show your work. Regardless of the method, you should report the answer values on this exam below.

- a. **(4 Points)** Compute the sample mean, sample variance, sample standard deviation, t-value, and half-lengths of the 95% confidence intervals for the expected time in system for each of the two alternative designs separately.

- b. **(5 Points)** You are really interested in the difference in performance between these two designs. That said, first perform a paired samples analysis on the data. Compute a 95% confidence interval on the expected value of the difference between Design A and Design B. In doing this, you should compute the mean difference, variance of the differences, standard error of the differences, and a critical t-value. You should also compute a t-statistic that can be compared with the critical t-value as a second method for determining if a statistically significant difference exists. Is there a statistically significant difference? Justify your answer.

- c. **(5 Points)** Now perform an unpaired analysis on the data. Compute a 95% confidence interval on the expected value of the difference as you did in part (b) only using an UNPAIRED analysis. Similar to part (b) you should compute the mean difference as well as the variances, a standard error, and a critical t-value necessary to perform the UNPAIRED analysis. You will again compute a t-statistic that can be compared with this critical t-value as a second method for determining if a statistically significant difference exists. Does the unpaired analysis show there a statistically significant difference?

- d. **(3 Points)** Do the results of your paired and unpaired analysis agree? Why do you think they did/didn't? Which method do you think is most appropriate in this situation?
- e. **(3 Points)** Determine the how many extra replications would be required to estimate the expected value of the difference in average time in the system for Design A vs. Design B with an absolute precision of ± 1.0 minute and with a confidence level of 95%? (Use the results from your paired t-test).

- f. **(3 Points)** Now consider Design B in more detail. Determine the how many extra replications would be required to estimate the expected value of the average time in the system for Design B with an absolute precision of ± 1.0 minute and with a confidence level of 95%?
- g. **(2 Points)** Comparing the resulting required number of replications in parts (e) and (f), in this application does simulation appear more effective in estimating the performance of a single system, or is simulation more effective in estimating difference in performance between alternative systems?

Problem 3. (50 Points Total) In a workstation with a single drilling machine for processing parts, the parts arrive randomly over time with a constant arrival rate of 1.25 parts per hour. The time to service a part has a probability density function (pdf) with the following functional form:

$$f_s(t) = \begin{cases} 0 & \text{if } t < 0 \text{ hr,} \\ Ct^2 & \text{if } 0 \leq t \leq 1 \text{ hr,} \\ 0 & \text{if } t > 1 \text{ hr,} \end{cases}$$

- a. **(10 Points)** Compute the mean and variance of the service time distribution given above. Compute the long-run (steady-state) utilization of the drilling machine and the long-run average waiting time in the queue for this system. (Hint: you may be able to verify the long-run average waiting time in the queue using a spreadsheet on the course website). Show your work for calculations.

b. **(10 Points)** Explain how to generate samples from the distribution by the method of inversion. Compute any functions you may need for your explanation and show your work in doing so.

c. **(5 Points)** Explain how the method of inversion works in your own words. You may want to draw a picture.

- d. **(10 Points)** Using Lindley's Recursion, set up a spreadsheet simulation of this workstation to estimate the expected value of the average time that the first 50 parts spend waiting for service. Base your estimate on 5 replications, and use a confidence level of 95%. Is the result that you get a good estimate of the long-run average waiting time? Explain why or why not.

You **MUST** complete the template in the Problem 3 tab of the ISE441Spring2012 - Midterm 1 - Spreadsheet Template.xlsx. Answer part (d) by hitting F9 several times and determining relatively how often the CI covers the steady-state value.

- e. **(5 Points)** Explain the Central Limit Theorem (CLT) briefly and in your own words (as short as 4-5 words will do). Also explain how we have structured simulations in class and on homework to make use of this theorem. (Hint: you should have actually applied this principle in creating your CI in your spreadsheet simulation in part d).

- f. **(10 Points)** Build a process-interaction Arena simulation model of the operation of the workstation, and simulate the first 50 parts to passing through the entire system. Make 20 replications of the simulation and report a 95% confidence interval on the expected value of the average time in the queue for the first 50 parts. Include the mean, half-width, lower limit, and upper limit. Does your 95% CI cover the expected steady-state waiting time? What could you change about your simulation model so that when you ran the 20 replications the 95% CI would almost certainly include the steady-state value? Make that change and report the new CI. Does the new CI cover the steady-state expected waiting time?
- g. **(5 Points) EXTRA CREDIT:** For the simulation in part (f) from 20 replications of 50 parts, Arena reported a 95% CI. Suppose you wanted a 90% CI. Use the information provided in the original CI to compute the half width of a 90% CI for the same performance measure.

You **MUST** submit your ISE441Spring2012 - Midterm 1 - Spreadsheet Template.xlsx file and Arena .doe file from Problem 3 in Moodle under the Midterm Exam 1 Assignment.