## ISE 441: Homework \#1, Fall 2011

## Due: Wednesday, January 25, 2012, at 11:55 p.m. for Moodle submission.

## PART I: Simulation By Hand

## Problem 1:

For the hand simulation of the simple processing system covered in class and presented in Section 2.4 in the book, define a new time persistent statistic: The total number of parts in the system, $N(t)$, which includes the parts in queue and in service. Augment Table 2-2 to track this as a new global variable. Add statistical accumulators to get its time-average and maximum [ $\left[\mathrm{S} N(t), N(t)^{*}\right]$, and compute these values at the end of the simulation.

Additionally, define another new time persistent statistic: $N_{2}(t)=Q(t)+1$, or simply the number of customers in the queue at time $t$ plus one. Augment Table 2-2 to track this as a new global variable as well. Add statistical accumulators to get its time-average and maximum [ $\left[N_{2}(t)\right.$, $\left.N_{2}(t)^{*}\right]$, and compute these values at the end of the simulation. Compare the performance measures to those computed for $N(t)$. Would it be appropriate to simply define $N(t)=Q(t)+1$ ? Why/Why not? What about defining $N(t) *=\max \{Q(t)\}+1$ ?

You will need to modify the Homework 1 Excel Spreadsheet Template - Problem 1 Tab. You will need to add several columns (in the appropriate places) to the hand simulation table and completely fill in the table including the new columns you have added. You also need to fill in the new performance measures you were asked for in the table (Revised Table 2-3) provided for you below the hand simulation table. The inter-arrival times and service times are available in the template for your convenience.
(Hint for all hand simulations problems that require revisions to the hand simulation in Table 2-2 of the book: You may be able to copy and paste some information from Table 2-2 in the book. In addition, if you are clever with formula writing, you will be able to get excel to keep track of the integrals and some of the other values in the hand simulation automatically, which makes your life easier)

## Problem 2:

Suppose you observe the following arrivals and departures from a single-server queueing system. Observations begin at time 100.0 (with no one in the queue) and end at 120.0 .

| Customer <br> $\#$ | Time Customer <br> Arrives to Queue | Time Customer Departs from <br> Queue and Goes Into Service | Time Customer <br> Departs From Server |
| :---: | :---: | :---: | :---: |
| 83 | 100.2 | 103.5 | 106.8 |
| 84 | 101.7 | 106.8 | 109.3 |
| 85 | 106.2 | 109.3 | 113.8 |
| 86 | 107.8 | 113.8 | 114.3 |
| 87 | 114.5 | 114.5 | 115.9 |
| 88 | 115.5 | 115.9 | 119.7 |
| 89 | 116.5 | 119.7 | After 120.0 |

1) What is the sample mean, sample standard deviation, minimum, and maximum of the waiting time in the queue observed for the customers who arrived at, and departed from, the queue during the time interval [100.0, 120.0]? You may use Excel formulas, but be sure they are correct!
2) What is the sample mean, sample standard deviation, minimum, and maximum of the total time in the system observed for the customers who arrived at, and departed from, the system during the time interval [100.0, 120.0]? You may use Excel formulas, but be sure they are correct!
3) What is the time-averaged (time-weighted) mean number of customers waiting in the queue during the observed time interval [100.0, 120.0]? Choose one of the following: show detailed calculations, show graph with clear labels and accompanying calculations, or use hand simulation table provided.
4) What is the time-averaged (time-weighted) total number of customers in the system during the observed time interval [100.0, 120.0]? Choose one of the following: show detailed calculations, show graph with clear labels and accompanying calculations, or use hand simulation table provided.
5) What is the utilization of the server during the observed time interval [100.0, 120.0]? Choose one of the following: show detailed calculations, show graph with clear labels and accompanying calculations, or use hand simulation table provided.

You need to use the Homework 1 Excel Spreadsheet Template - Problem 2 Tab. If you chose to do this by hand, it must be extremely neat, and you will need to scan both of your labeled graphs and submit them as PDF files.

## Problem 3:

For the hand simulation of the simple processing system covered in class and presented in Section 2.4 in the book, suppose that up to time 10.0, only one drill press and drill press operator are available to process parts; but starting at time 10.0, there are two drill-press operators and two drill presses that are available to work on parts simultaneously and independently of each other. Each individual part has the same arrival time and the same processing time as in the original system (both drills are the same speed), but now processing of the part may be on either of the two drill presses that become free when the part reaches the front of the queue.

In addition to all the other performance measures of interest (Table 2-3), we seek to compute the following time-persistent (time-weighted) statistics:
(a) The time-averaged number of busy drill presses;

Redefine $B(t)$ = the number of drill presses busy at time $t$ (now $0 \leq B(t) \leq 2$ ).
Time Averaged Number of Bust Drill Presses $=\frac{\int_{0}^{T} B(t) d t}{T}$
(b) The time-averaged number of drill presses that are available;

Define $B_{A}(t)=$ the number of drill presses available at time $t$
Time Averaged Number of Drill Presses Available $=\frac{\int_{0}^{T} B_{A}(t) d t}{T}$
(c) The drill press utilization, which is usually defined to be (a) divided by (b).

$$
\text { Drill Press Utilization }=\frac{\int_{0}^{T} B(t) d t}{\int_{0}^{T} B_{A}(t) d t}
$$

Note carefully that one drill press is available from time 0.0 up to time 10.0; and then starting at time 10.0 and going up to time 20.0, there are two drill presses available. Compute the values of all the relevant observation-based and time-persistent statistics over the time period [0, 20.0].

What is the utilization of the drill presses and how does this consideration change the originally computed performance measures?

You will need to modify the Homework 1 Excel Spreadsheet Template - Problem 3 Tab. You will need fill in the hand simulation table completely. Again, the inter-arrival times and service times are also in the template for your convenience.

You need to fill in: (a) the revised Table 2-3, (b) the original Table 2-3, and (c) the table that which subtracts the values in (a) from those in (b), all of which are provided for you below the hand simulation table. Discuss how making this new drill press available changes the performance measures. Again, you may use formulas, but be sure they are correct.

## Problem 4:

For the hand simulation of the simple processing system covered in class and presented in Section 2.4 in the book, suppose that the scheduling discipline were changed so that when the drill press becomes idle and finds parts waiting in the queue, instead of taking the first one (FIFO), it takes the one that will require the shortest processing time (this is usually called the SPT discipline). To make this work, you will need to assign an attribute to parts in when they arrive, representing what their service time at the drill press will be. (Hint: instead of listing the arrival time of the parts in service and in the queue in the attributes columns, you might be wise to list the service time...) Re-do the hand simulation and compute the values of all the relevant observation-based and time-persistent statistics over the time period [0, 20.0]. Is this a better rule in terms of some of the performance measures? If so, which ones?

You will need to modify the Homework 1 Excel Spreadsheet Template - Problem 4 Tab. You will need fill in the hand simulation table completely. Again, the inter-arrival times and service times are also in the template for your convenience.

You need to fill in: (a) the revised Table 2-3, (b) the original Table 2-3, and (c) the table that which subtracts the values in (a) from those in (b), all of which are provided for you below the hand simulation table. Discuss how changing the queuing discipline changes the performance measures. Again, you may use formulas, but be sure they are correct.

## Problem 5:

For the hand simulation of the simple processing system covered in class and presented in Section 2.4 in the book, suppose that the following changes were made to the system:

1. There are now two drill presses for the entire 20 minutes of run time, however
a. One of the drill presses goes down for maintenance from time 5.0 to time 10.0
2. Each part requires a two-minute setup time before it can be processed
3. The scheduling discipline is SPT

To make this work, you will need to combine what you have learned how to do in your previous augmentations. You will need to take into account there are multiple resources some of the time, each part requires setup, and the new scheduling discipline. Re-do the hand simulation and compute the values of all the relevant observation-based and time-persistent statistics over the time period [0, 20.0].

You will need to modify the Homework 1 Excel Spreadsheet Template - Problem 5 Tab. You will need to add several columns (in the appropriate places) to the hand simulation table and completely fill in the table including the new columns you have added. You also need to fill in the new performance measures you were asked for in the table (Revised Table 2-3) provided for you below the hand simulation table. Again, the inter-arrival times and service times are also in the template for your convenience.

You need to fill in the revised Table 2-3 provided for you below the hand simulation table. Again, you may use formulas, but be sure they are correct.

## PART II: Confidence Intervals for Differences and Statistical vs. Practical Significance

## Problem 6:

Two alternative designs have been proposed for processing customers in a service center. To compare the performance of the two alternative system designs, we generated 10 days of customer traffic, where day's traffic consists of a new simulated sequence of customer arrival times and the associated service times. For the first day of simulated operation (that is, on simulation replication 1) under design A, we ran the traffic for day 1 through the system operating under design A to compute the average time in system for all the customers processed on that day (replication). Then for the first day (replication) of simulated operation under design B , we ran the same traffic for day 1 through the system operating under design B to compute the average time in system for all the customers processed on that day (replication). This process of making paired replications of system operation under designs A and B was repeated for a total 10 days (independent replications) of system operation under each alternative design. We computed the average time in system (in minutes) for all the customers processed on each day as follows:

| Replication No. |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average Time in System (Minutes) | Design A | 135.1 | 205.1 | 208.2 | 141.7 | 254.3 | 164.1 | 266.9 | 127.2 | 128.9 | 171.2 |
|  | Design B | 105.1 | 199.3 | 170.2 | 109.1 | 211.2 | 172.7 | 224.4 | 112.0 | 92.3 | 131.6 |

1) Compute the sample mean, sample standard deviation, and half-widths for both $95 \%$ and $99 \%$ confidence intervals on the expected value of the average time in system for each of the two alternative policies.
2) Based on the paired differences between the average time in system for Policy A and the average time in system for Design B on each replication, compute the sample mean, sample standard deviation, and half-length of a $95 \%$ confidence interval for the difference between the expected time in system for Design A and the average time in system for design B.
3) Based on your analysis of the paired differences in performance on each replication, is there a statistically significant difference between the expected time in system for the two alternative designs at the $5 \%$ level of significance? You must justify your answer by solving this problem two different ways: (1) Examine the confidence interval of the difference and make a statement based upon the upper and lower limits; and (2) Compute the sample t -value $\left(\mathrm{t}_{0}\right)$ and the critical t -value ( $\mathrm{t}_{\text {crit }}$ ) and make a statement based on a comparison of these two values.
4) Do you regard the difference in average system time between the two alternative designs to be practically significant? Justify your answer. (Hint: Take a second to compare the values for each replication individually... do you notice something...?)

You will need to modify the Homework 1 Excel Spreadsheet Template - Problem 6 Tab. Places for each required response have been provided, however you may still want to add some rows and columns to the table of data given in order to do your computations. Again, you may use formulas.

Please also answer the administrative questions in the final tab of the template.

## For this Homework you will submit:

1. One Excel File (the spreadsheet template) with your full name and HW 1 in the filename.
2. If necessary, any PDF files which have calculations or graphs you chose to do by hand, though I encourage you to use excel, it can help you do minimize the time needed to complete the assignment.
