ISE 441 – Introduction to Simulation
In Class Quiz #3b (Full Version)
1/23/12

NAME _____Solution__________________________

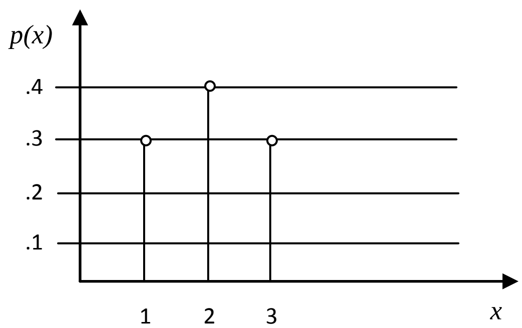
THIS QUIZ IS CLOSED BOOK, CLOSED NOTES, OPEN CALCULATOR

You should pull up t-table from the course website and keep it on the screen for the duration of the quiz. Visiting other websites will result in a zero for this quiz. The screens are being monitored at the podium computer.

1. Suppose *X* is a discrete random variable with probability mass function given by:

   P{*X* = 1} = 0.3
   P{*X* = 2} = 0.4
   P{*X* = 3} = 0.3

   a. Plot the PDF of *X* for all *x* in the range

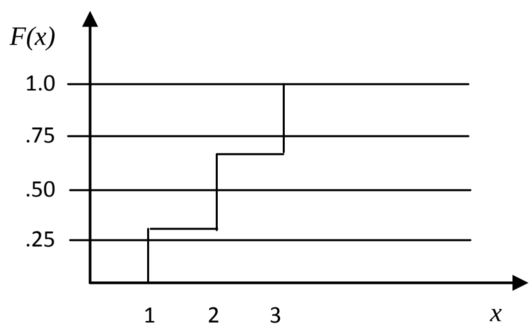



   b. Compute and plot the CDF of *X* for all *x* in the range

   **P{*X* ≤ 1} = 0.3**

   **P{*X* ≤ 2} = 0.7**

   **P{*X* ≤ 3} = 1.00**



c. Compute $P\{1.5 \leq X \leq 3\}$

$P\{X \leq 3\} - P\{X \leq 1.5\} = 1.00 - 0.3 =$ **0.7**

d. Compute $P\{1.5 < X < 3\}$

$P\{X < 3\} - P\{X < 1.5\} = 0.7 - 0.3 =$ **0.4**

e. Compute $E[X]$

$E[X] = 0.3(1) + 0.4(2) + 0.3(3) =$ **2.0**

f. Compute the Variance and Standard Deviation of $X$

Method 1:

$Var[X] = [1^2(0.3) + 0^2(0.4) + 1^2(0.3)] =$ **0.6**

$StDev[X] = SQRT(Var[X]) =$ **0.775**

Method 2:

$E[X^2] = 0.3(1^2) + 0.4(2^2) + 0.3(3^2) = 4.6$

$Var[X] = E[X^2] - (E[X])^2 = 4.6 - 2^2 =$ **0.6**

$StDev[X] = SQRT(Var[X]) =$ **0.775**

2. Suppose $X$ has a continuous distribution where x has a minimum of 0, a maximum of 5, and the following PDF: $f(x) = Cx^2$.

a. Compute the Constant C that makes this a valid PDF

$$C\int_0^5 x^2 dx = 1 \qquad C\left[\frac{x^3}{3}\right]_0^5 = 1 \qquad C\left[\frac{5^3}{3}\right] = 1 \qquad C = \frac{3}{5^3} = \frac{3}{125} = 0.024$$

b. Compute P{$X \le 3$}

$$P(X \le 3) = 0.024\int_0^3 x^2 dx = 0.024\left[\frac{x^3}{3}\right]_0^3 = 0.024\left[\frac{3^3}{3}\right] = 0.216$$

c. Compute P{$X < 3$}

$$P(X < 3) = P(X \le 3) = 0.216$$

d. Compute P{$X > 3$}

$$P(X > 3) = 1 - P(X < 3) = 1 - 0.216 = 0.784$$

e. Compute P{$X < 2$ or $X > 4$}

$$P(X < 2 \text{ or } X > 4) = \left[0.024\int_0^2 x^2 dx\right] + \left[0.024\int_4^5 x^2 dx\right] =$$

$$\left\{0.024\left[\frac{2^3}{3}\right]\right\} - \left\{0.024\left[\frac{5^3}{3}\right] - 0.024\left[\frac{4^3}{3}\right]\right\} = 0.064 + [1 - 0.512] = 0.552$$

f. Compute $E[X]$

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx = 0.024\int_0^5 x^3 dx = 0.024\left[\frac{x^4}{4}\right]_0^5 = 0.024\left[\frac{5^4}{4}\right] = 3.75$$

3. Suppose that 6.5, 6.2, 8.8, 7.6, 8.9, 10.1, 9.0, 10.0, 6.0 & 6.9 are ten observations from a Normal Distribution with unknown mean and unknown variance. Compute the sample mean, sample variance, and 90%, 95%, and 99% two-sided CI's on the population mean. You may use the link to the *t*-tables posted on the course website.

Mean = (6.5 + 6.2 + 8.8 + 7.6 + 8.9 + 10.1 + 9.0 + 9.5 + 7.2 + 6.9) / 10 = **8.0**

Variance = $(1.5^2 + 1.8^2 + 0.8^2 + 0.4^2 + 0.9^2 + 2.1^2 + 1^2 + 2^2 + 2^2 + 1.1^2)$ / 9 = **2.413**

StDev = 1.553

$t_{0.05, 9} = 1.833$ $t_{0.025, 9} = 2.262$ $t_{0.005, 9} = 3.250$

90% CI = 8.0 ± 1.833*[1.553/SQRT(10)] = **8.0 ± 0.9 = [7.1, 8.9]**

95% CI = 8.0 ± 2.262*[1.553/SQRT(10)] = **8.0 ± 1.11 = [6.89, 9.11]**

99% CI = 8.0 ± 3.250*[1.553/SQRT(10)] = **8.0 ± 1.60 = [6.4, 9.6]**